'Some probabilistic ideas in coastal morphodynamics - applications to flood defence and offshore banks'

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Flood Risk

Risk = Hazard x Consequence
Coastal Flooding

‘Flooding is an issue that affects us all. Over £200 billion worth of assets are at risk around British rivers and coasts and these risks are likely to increase over the next 100 years due to changes in climate and in society.’

Flood defence or coastal protection?
Causes of Coastal Flooding
Predicting wave overtopping volumes

- Goda
- Van der Meer
- Owen

Courtesy of HR Wallingford
Owen’s overtopping formula

\[ Q_m = g H_s T_m A e^{-(BR_c/rT_m\sqrt{gH_s})} \]
Beaches are mobile

Before

After!!
What about morphology?

What if the beach level goes down?

1. Water depth increases
2. So $H_b$ increases and so $Q$ increases
Background

What is coastal morphology and why is it important?

• Beaches; sandbanks, channels
• Protects coastal infrastructure
• Affects port development, dredging, marine renewables installations
A simple model of beaches

- The ‘1-line’ model
  - Developed from laboratory studies in 1950’s
  - Describes the evolution of the shape of a single beach level contour
  - Under ‘small angle’ assumptions simplifies to a diffusion type equation
    - Analytical solutions for constant wave conditions (1950- )
    - Analytical solutions for time varying waves (1997- )
    - Solutions for moments (2004- )
  - In general requires numerical solution of 3 equations
    - Monte Carlo solutions – specification of wave stats!!
A simple model of beaches

1. ’Constant’ beach profile shape

2. Sand transported by action of breaking waves

3. Profile movement between upper and lower limits
A simple model of beaches

- Distance Offshore
- Project Lateral Boundary
- Shoreline Coordinate
- Shoreline
- Distance Alongshore
- Trend of Offshore Bottom Contours
- Longshore Grid Spacing
- Project Lateral Boundary
Simple models

\[ \frac{\partial y}{\partial t} = \frac{1}{D} \frac{\partial Q}{\partial x} \]  \hspace{1cm} (1)

\[ Q = Q_0 \sin(2\alpha_b) \]  \hspace{1cm} (2)

\[ \alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right) \]  \hspace{1cm} (3)

\[ Q_0 = \frac{\kappa P}{2g(\rho_s - \rho)(1 - p_s)} = \frac{\rho}{16} H_b^2 c_{gb} \frac{\kappa}{(\rho_s - \rho)(1 - p_s)} \]
1-line model - solutions

Straight impermeable groyne

Boundary conditions:

\[ Q = 0 \text{ at } x = 0, \text{ so } \tan(\alpha_b) = \frac{dy}{dx} \]
\[ y(x,t) \text{ tends to zero for large } x \]

\[ \frac{\partial y}{\partial t} = K \frac{\partial^2 y}{\partial x^2} \]

Solution

\[ y(x,t) = \tan \alpha_0 \sqrt{\frac{4Kt}{\pi}} \left\{ e^{-\frac{x^2}{4Kt}} - \frac{x\sqrt{\pi}}{2\sqrt{Kt}} \text{erfc} \left( \frac{x}{2\sqrt{Kt}} \right) \right\} \]

Erfc(x) = 1-erf(x) is the complementary Error function. Erf(x) is the Error function.
1-line model - solutions
Beaches in a groyne field
Uncertainty

Simple model worked OK – but not perfect

• Varying waves
• Varying sediments

• Treat uncertainty through stochastic approach
Beach model revisited

Options

• Solve equations for the statistics (Moment equations) – difficult

• Perform Monte Carlo simulation with your model of choice – calculate *sample* statistics from the outputs.
Practical applications

• Increasing adoption of ‘Soft engineering’ options that include beach/dune/saltmarsh elements

• Uncertainties in performance of schemes that rely on beaches

• Uncertainties in loading due to climate change

• Detached breakwater schemes?
Current guidance on designing offshore breakwaters

- CIRIA’s Beach Management Manual (1996) – ‘… detached breakwaters can induce very strong changes in the morphodynamic regime…’

- Fleming & Hamer (2000) – ‘…. the existing design guidance was inadequate to distinguish actual response…’

- CEM (2003) – ‘…. Optimizing detached breakwater designs is difficult when large water level variations are present,…’
Motivation & Background
Field response: ■=Tombolo, △=Salient, ×=No Sinuosity
Predicted response: Dean & Pope (1986)
Aim

• Improve understanding of detached breakwater schemes in macro tidal environments

• Develop an appropriate modelling methodology to analyse the uncertainties in scheme response arising from natural inter-annual wave variability

• Use a real scheme, experimental data and numerical modelling

• Part of the LEACOAST 2 project (EPSRC)
Experimental data – the site
Experimental data

- Monthly topographic and bathymetric surveys
- Tides, waves, currents, sediments
Modelling approach

One-line beach model with 5 enhancements:

- new solution method based on ‘method of lines’;
- dynamic linking with sophisticated wave model;
- accounting for tidal currents;
- accounting for the effects of overtopping;
- running the whole in Monte Carlo mode.

\[
\frac{\partial y}{\partial t} = \frac{1}{D_c} \frac{\partial Q}{\partial x} + q \quad (1)
\]

\[
Q = Q_0 \sin(2\alpha_b) \quad (2)
\]

\[
\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right) \quad (3)
\]
Model calibration (deterministic)

General NW-SE drift of 154,000 m$^3$/year ($K_1 = 1.0$). Significant annual variation. Agrees well with previous studies e.g. Vincent (1979).
Model calibration (stochastic)

Importance of recreating the correct statistical properties.

We have used a method described by Cai (2005) and Cai et al. (2007) to create multiple realisations of 13-year time series; the statistical properties mirror those in the original time series of hindcast waves well.
Monte Carlo results

A single realisation from 200:
Bay 1 – annual cycle + isolated significant events
Bay 2 – long term drift
Monte Carlo results

Envelope of beach position over 200 realisations and 13 years

Bays 1-3 exhibit greatest range of variation

Morphological features (tombolos, salients) appear stable
Summary of beach modelling

- DEFRA are encouraging the use of ‘probabilistic design’

- Limited ‘tools’ to do this

- Monte Carlo modelling beyond @RISK-type spreadsheet simulation is limited to research groups at present

- Analytical and numerical work is being conducted by several groups internationally
Ports and sandbanks

Drivers: New port development – how does mobility of sandbanks affect dredging and navigation?

Study site is Gt. Yarmouth and surroundings. Curved shoreline and a group of mobile sandbanks can be found. The dataset comprises high quality historical survey charts dating back to the 19th century.

A data-driven model has been developed to analyse the long-term evolution of a sandbank system and to make ensemble prediction over a period of 8 years.
Motivation

- Good areas for wind farms:
  - 30 2MW wind turbines
  - Monopile, D=4.2m, L=50m,
  - weight=200t
- Support tern and seal colonies...
- and also tourism.
Questions

• Presence of Wind farm: Motion of Scroby Sands Sandbanks away from Foundations?

• Outer Harbour expansion: Long term evolution of sandbanks and navigational channels?

Photo Courtesy of Mike Page
Forecasting sandbanks

Long term evolution over 150 yrs
Ensemble predictions and uncertainties in a 8 year forecast.
Gt Yarmouth Sandbanks

Elevation (m CD)

0 m
5 m
10 m
15 m
20 m
25 m
30 m
35 m
40 m
45 m

1846 1864 1875 1886 1896 1905 1916 1922 1934
Methodology

• Combine:
  - EOF Analysis: spatial & temporal patterns
  - Jack-knife resampling: generate an ensemble
  - Causal auto-regression:
    • To extrapolate the temporal eigenfunctions.
  - Statistical analysis of ensemble forecasts
**Methodology**

**EOF technique**

Discrete beach or seabed levels by \( g(l, t_k) \), with \( 1 \leq l \leq L \) and \( 1 \leq k \leq K \)

Expand \( g \):

\[
g(l, t_k) = \sum_{p=1}^{L} \alpha_p c_p(t_k) e_p(l)
\]

\( e_p(l) \) -- eigenvectors of the data’s correlation matrix

Eigenvalues – Variance resolved, measure of energy

\( c_p(t_k) \) -- temporal eigenfunctions.

**Creating the ensemble**

Ensemble produced using the Jack-Knife technique:

a) Remove a bathymetry from the set of 33

b) Perform interpolation and EOF analysis of resulting sample between 1848 and 1998

c) Compute the mean of the ensemble and use it as the sample statistic.

**Ensemble Forecast:**

Forecasts between 1999 and 2006

**Burg’s algorithm**

\[
c_p(t_k) = -\beta_1 c_p(t_{k-1}) - \beta_2 c_p(t_{k-2}) - \ldots - \beta_r c_p(t_{k-r}) + \varepsilon_p(t_k)
\]

**Least Square Auto-Regressive algorithm**

\[
f(\beta) = \sum_{t_k=r+1}^{N} |e(t_k)|^2 = \sum_{t_k=r+1}^{N} |c_p(t_k) + \varphi^T \beta|^2
\]
Results

**Ensemble of EOFs**

$c(t)$ for the ensemble of jack-knifes from the dataset (all values shown including the control)

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**Forecast of Temporal EOFs**

Absolute $rmse$ of the forecasts relative to the measured EOFs, in the medium (a) and long-term (b). Forecasts were obtained with Burg’s and LSAR algorithm of order $r=3$ (solid line), $r=5$ (dashed line) and $r=50$ (thin line with dots). The left and right panels correspond to the $rmse$ for the first three and first eight EOFs, respectively.

$$rmse(t) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} [C_{i,f}(t) - C_{i,m}(t)]^2}$$
Results

**EOF Analysis**

Bathymetric reconstruction in shading maps elevations (in metres) for each of the first three spatial eigenfunctions. The first EOF represents the mean bathymetry, while the second and third EOFs resolve variations about the mean.
Results

Forecasts to year 2006 from year 1998.

a) Bathymetry of 2006,

Mean bathymetry of the ensemble forecasts using:
b) Burg’s Algorithm
c) LSAR Algorithm

Standard deviation between measurements and predictions using:
d) Burg’s Algorithm
e) LSAR Algorithm

Skewness between measurements and predictions using:
f) Burg’s Algorithm
g) LSAR Algorithm
Conclusions

- Predicting coastal change is hard!
- Probabilistic methods are being developed
- Encouraging results but...
  - How to present answers
  - How to improve design

List of publications available:
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Thank you for your attention!